AP Calculus AB
Section 6.5 – Worksheet KEY

Name: ______________________
Date: __________ Block: ________

Evaluate the following definite integrals.

1. $\int_{-1}^{2} (x^3 - 2x)\, dx$

$= \left(\frac{x^4}{4} - \frac{2x^2}{2}\right)\Big|_{-1}^{2} = \left(\frac{x^4}{4} - x^2\right)\Big|_{-1}^{2}$

$= \left(\frac{(2)^4}{4} - (2)^2\right) - \left(\frac{(-1)^4}{4} - (-1)^2\right)$

$= \left(\frac{16}{4} - 4\right) - \left(\frac{1}{4} - 1\right) = (4-4) - \left(-\frac{3}{4}\right) = 0 + \frac{3}{4} = \frac{3}{4}$

2. $\int_{1}^{8} \sqrt[3]{x}\, dx$

$= \int_1^8 x^{\frac{1}{3}}\, dx = \frac{x^{\frac{4}{3}}}{\frac{4}{3}}\Bigg|_1^8 = \frac{3}{4} x^{\frac{4}{3}}\Big|_1^8$

$= \frac{3}{4}(8)^{\frac{4}{3}} - \frac{3}{4}(1)^{\frac{4}{3}} = \frac{3}{4}(16) - \frac{3}{4}(1) = 12 - \frac{3}{4}$

$= \frac{45}{4}$

3. $\int_{\frac{\pi}{6}}^{\pi} \sin x\, dx$

$= -\cos x|_{\frac{\pi}{6}}^{\pi} = -\cos(\pi) - -\cos\left(\frac{\pi}{6}\right)$

$= -\cos(\pi) + \cos\left(\frac{\pi}{6}\right)$

$= (-1) + \left(\frac{\sqrt{3}}{2}\right) = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$

4. $\int_{0}^{\frac{\pi}{4}} \sec\theta \tan\theta\, d\theta$

$= \sec\theta|_0^{\frac{\pi}{4}} = \sec\left(\frac{\pi}{4}\right) - \sec(0) = \sqrt{2} - 1$

5. $\int_{1}^{9} \frac{x-1}{\sqrt{x}}\, dx$

$\int_1^9 \frac{x-1}{x^{\frac{1}{2}}}\, dx = \int_1^9 \left(\frac{x}{x^{\frac{1}{2}}} - \frac{1}{x^{\frac{1}{2}}}\right) dx = \int_1^9 \left(x^{\frac{1}{2}} - x^{-\frac{1}{2}}\right) dx$

$= \left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right)\Bigg|_1^9 = \left(\frac{2}{3}x^{\frac{3}{2}} - 2x^{\frac{1}{2}}\right)\Big|_1^9$

$= \left(\frac{2}{3}(9)^{\frac{3}{2}} - 2(9)^{\frac{1}{2}}\right) - \left(\frac{2}{3}(1)^{\frac{3}{2}} - 2(1)^{\frac{1}{2}}\right)$

$= \left(\frac{2}{3}(27) - 2(3)\right) - \left(\frac{2}{3}(1) - 2(1)\right)$

$= (18 - 6) - \left(\frac{2}{3} - 2\right) = 12 - \left(-\frac{4}{3}\right) = \frac{40}{3}$

6. $\int_{0}^{1} (x+2)(x-3)\, dx$

$\int_0^1 (x^2 - x - 6)\, dx = \left(\frac{x^3}{3} - \frac{x^2}{2} - 6x\right)\Big|_0^1$

$= \left(\frac{(1)^3}{3} - \frac{(1)^2}{2} - 6(1)\right) - \left(\frac{0^3}{3} - \frac{0^2}{2} - 6(0)\right)$

$= \left(\frac{1}{3} - \frac{1}{2} - 6\right) - (0 - 0 - 0)$

$= -\frac{37}{6}$

7. $\int_1^4 2x^{-1}dx$

$2\ln|x||_1^4 = 2\ln(4) - 2\ln(1)$
$= 2\ln(4) - 2(0) = 2\ln(4)$

8. $\int_0^2 (e^x + e)\,dx$

$= (e^x + ex)|_0^2$
$= \left(e^2 + e(2)\right) - \left(e^0 - e(0)\right)$
$= (e^2 + 2e) - (1 - 0) = e^2 + 2e - 1$

9. $\int_0^3 (2\sin x - e^x)dx$

$= (-2\cos x - e^x)|_0^3$

$= (-2\cos(3) - e^3) - (-2\cos(0) - e^0)$

$= (-2\cos(3) - e^3) - (-2(1) - 1)$
$= (-2\cos(3) - e^3) - (-3)$
$= -2\cos(3) - e^3 + 3$

10. $\int_1^2 \frac{v^3 + 3v^6}{v^4}\,dv$

$= \int_1^2 \left(\frac{v^3}{v^4} + \frac{3v^6}{v^4}\right) dv = \int_1^2 \left(\frac{1}{v} + 3v^2\right) dv$
$= \int_1^2 (v^{-1} + 3v^2)\,dv = \left(\ln|v| + \frac{3v^3}{3}\right)\Big|_1^2$
$= (\ln|v| + v^3)|_1^2$
$= (\ln(2) + (2)^3) - (\ln(1) + (1)^3)$
$= (\ln(2) + 8) - (0 + 1)$
$= (\ln(2) + 8) - (1) = \ln(2) + 7$

11. $\int_0^2 e^{x+3}dx$

$= \int_0^2 e^3 e^x\,dx = e^3 e^x|_0^2 = (e^3 e^2) - (e^3 e^0) = e^5 - e^3$

12. $\int_1^{18} \sqrt{\frac{3}{x}}\,dx$

$= \int_1^{18} \sqrt{3x^{-1}}\,dx = \int_1^{18} \sqrt{3}\sqrt{x^{-1}}\,dx$
$= \int_1^{18} \sqrt{3}x^{-\frac{1}{2}}\,dx = \frac{\sqrt{3}x^{\frac{1}{2}}}{\frac{1}{2}}\Bigg|_1^{18} = 2\sqrt{3}x^{\frac{1}{2}}\Big|_1^{18}$
$= 2\sqrt{3}(18)^{\frac{1}{2}} - 2\sqrt{3}(1)^{\frac{1}{2}}$
$= 2\sqrt{3}\left(3\sqrt{2}\right) - 2\sqrt{3}$
$= 6\sqrt{6} - 2\sqrt{3}$